Expansion of Determinants

1 Mark Questions

1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

Delhi 2014

Firstly, expand both determinants, which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x^2 - 40 = 18 - (-14)$

$\Rightarrow \quad 2x^2 - 40 = 18 + 14$

$\Rightarrow \quad 2x^2 - 40 = 32$

$\Rightarrow \quad 2x^2 = 32 + 40$

$\Rightarrow \quad 2x^2 = 72 \Rightarrow x^2 = 36$

$\therefore \quad x = \pm 6$ **(1)**

2. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, then find the value of x. All India 2014

Given, $$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$

On expanding the determinant of both sides, we get

$$3x \times 4 - (-2) \times 7 = 8 \times 4 - 6 \times 7$$

$$\Rightarrow \quad 12x - (-14) = 32 - 42$$

$$\Rightarrow \quad 12x + 14 = -10$$

$$\Rightarrow \quad 12x = -10 - 14$$

$$\Rightarrow \quad 12x = -24 \quad \Rightarrow \quad x = -\frac{24}{12}$$

$$\therefore \quad x = -2$$

(1)

3. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k. Foreign 2014

We know that, if A is a square matrix of order n. Then, $|kA| = k^n|A|$.

Here, the matrix A is of order 3.

$$\therefore \quad |3A| = (3)^3|A| = 27|A|$$

On comparing with $k|A|$, we get **(1)**

$$k = 27$$

4. Find (adj A), if $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$.

Delhi 2014C

Given, $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1$

We know that, $|\text{adj}(A)| = |A|^{n-1}$, where n is order of determinant.

$\therefore \quad |\text{adj}(A)| = |1|^{2-1} \Rightarrow |\text{adj}(A)| = 1$ **(1)**

5. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$.

Delhi 2014C

Suppose $A = \begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$

On expanding, we get

$$A = p^2 - (p-1)(p+1)$$

$$\Rightarrow \quad A = p^2 - (p^2 - 1^2)$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow \quad A = p^2 - p^2 + 1$$

$$\therefore \quad A = 1$$

6. If A is a square matrix of order 3 such that $|\text{adj}(A)| = 64$, then find $|A|$. **Delhi 2013C**

We know that, for a square matrix of order n, $|\text{adj}(A)| = |A|^{n-1}$ have $n = 3$

$$\therefore \quad |\text{adj}(A)| = |A|^{3-1} = |A|^2$$

Given, $|\text{adj}A| = 64$, $64 = |A|^2 \Rightarrow (8)^2 = |A|^2$

$\Rightarrow \quad |A| = \pm 8$ [taking square root] **(1)**

7. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then find the value of x. **Delhi 2013C**

> Expand both determinants which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$

$\Rightarrow 2x(x+1) - (x+3)(2x+2) = 3 - 15$

$\Rightarrow 2x^2 + 2x - (2x^2 + 8x + 6) = -12$

$\Rightarrow \quad -6x - 6 = -12$

$\Rightarrow \quad 6x = 6$

$\therefore \quad x = 1$ **(1)**

8. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. **Delhi 2013**

Given, $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2)-(x-3)(x-1)=12+1$

$\Rightarrow (x^2+3x+2)-(x^2-4x+3)=13$

$\Rightarrow \quad 7x-1=13$

$\Rightarrow \quad 7x=14$

$\therefore \quad x=2$ **(1)**

9. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$. **All India 2013; HOTS**

Let $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Here, $a_{32} = 5$

Given, A_{ij} is the cofactor of the element a_{ij} of A.

Then, $A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$

$= (-1)^5 (8-30) = -(-22) = 22$

$\therefore \quad a_{32} \cdot A_{32} = 5 \times 22 = 110$ **(1)**

10. Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$. **All India 2012**

We know that, for a square matrix A of order n,

$$|kA| = k^n \cdot |A|$$

Here, $|2A| = 2^3 \cdot |A|$ [$\because$ order of A is 3×3]

$= 2^3 \times 4 = 8 \times 4 = 32$ [$\because$ put $|A| = 4$] **(1)**

11. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor of the element a_{23}. Delhi 2012

Minor of the element

$a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$ **(1)**

12. If the determinant of matrix A of order 3×3 is of value 4, then write the value of $|3A|$.

All India 2012C

Given, the order of matrix A is 3×3 and

$|A| = 4$

$\Rightarrow |3A| = 3^3 \cdot |A|$ [$\therefore |KA| = K^n |A|$]

$= 3^3 \cdot 4 = 27 \cdot 4 = 108$ **(1)**

13. For what value of x, $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is a singular matrix? All India 2011C

For a singular matrix, $|A| = 0$. Use this relation and solve it.

Matrix A is said to be singular, if $|A| = 0$

$$\therefore \quad \begin{vmatrix} 2x+2 & 2x \\ x & x-2 \end{vmatrix} = 0$$

$$\Rightarrow \quad (2x+2)(x-2) - 2x^2 = 0$$

$$\Rightarrow \quad 2x^2 - 2x - 4 - 2x^2 = 0 \quad \Rightarrow \quad -2x = 4$$

$$\therefore \quad x = -2 \qquad \textbf{(1)}$$

14. For what value of x, the matrix $\begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$ is a singular matrix? **All India 2011C**

Let $A = \begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$

If matrix A is singular, then

$$|A| = 0$$

$$\therefore \quad \begin{vmatrix} 2x+4 & 4 \\ x+5 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad (2x+4) \times 3 - (x+5) \times 4 = 0$$

$$\Rightarrow \quad 6x + 12 - 4x - 20 = 0 \quad \Rightarrow \quad 2x = 8$$

$$\therefore \quad x = 4 \qquad \textbf{(1)}$$

15. For what value of x, the matrix $\begin{bmatrix} 2x & 4 \\ x+2 & 3 \end{bmatrix}$ is a singular matrix? **Delhi 2011C**

Do same as Que. 14. **[Ans.** $x = 4$]

16. For what value of x, matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix? **Delhi 2011C**

Do same as Que. 14. **[Ans.** $x = 2$]

17. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular? **Delhi 2011**

Do same as Que. 14. **[Ans.** $x = 3$]

18. Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$.

All India 2011; HOTS

Firstly, expand the determinant and use the trigonometric relation to calculate the value of determinant.

$$A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$$

On expanding, we get

$$A = (\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ)$$

$$= \cos(15^\circ + 75^\circ)$$

$[\because \cos x \cos y - \sin x \sin y = \cos(x+y)]$

$$= \cos 90^\circ = 0 \quad [\because \cos 90^\circ = 0] \text{ (1)}$$

19. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, then write the positive value of x. **Foreign 2011; All India 2008C**

Expand both determinants which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

On expanding, we get

$$x^2 - x = 6 - 4$$

$$\Rightarrow \quad x^2 - x - 2 = 0 \quad \Rightarrow \quad (x-2)(x+1) = 0$$

$$\therefore \quad x = 2 \text{ or } -1$$

Hence, the positive value of x is 2. **(1)**

20. What is the value of determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$? **Delhi 2010**

Determinant can be easily expand either corresponding to a row or column which have maximum zeroes.

Given, determinant

$$A = \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$$

$$\Rightarrow \quad |A| = -2\,(12 - 16)$$

[$\because$ expanding along R_1]

$$= -2\,(-4) = 8$$ **(1)**

21. Find the minor of the element of second row and third column in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$. **Delhi 2010**

Minor of the element of second row and third column is given by

$$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13 \quad \textbf{(1)}$$

22. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then find $|\text{adj } A|$.

Delhi 2010C; HOTS

Given, $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$

Cofactors of A are

$C_{11} = -3, C_{12} = -2, C_{21} = -1, C_{22} = 3$

We know that, adjoint $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} -3 & -2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\text{Now } |\text{adj}(A)| = \begin{vmatrix} -3 & -1 \\ -2 & 3 \end{vmatrix} = -3 \times 3 - (-1 \times -2)$$

$$= -9 - 2 = -11$$

$$\Rightarrow \quad |\text{adj}(A)| = -11 \quad \textbf{(1)}$$

Alternate Method

Here, $|A| = \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -9 - 2 = -11$

Using the result

$$|\text{adj}(A)| = |A|^{n-1}$$

where, n is order of a determinant, we get

$$|\text{adj}(A)| = (-11)^{2-1} = -11 \quad \textbf{(1)}$$

23. If $|A| = 2$, where A is a 2×2 matrix, then find $|\text{adj } A|$.

All India 2010C

Given, $|A| = 2$, where A is a 2×2 matrix.

We know that, $|\text{adj}(A)| = |A|^{n-1}$, where n is the order of matrix. Here, we have

$$n = 2 \text{ and } |A| = 2$$

$\therefore$ $|\text{adj}(A)| = (2)^{2-1}$

$\Rightarrow$ $|\text{adj}(A)| = 2$ **(1)**

24. What positive value of x makes following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

All India 2010

Given, $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

On expanding, we get

$$2x^2 - 15 = 32 - 15 \Rightarrow 2x^2 - 15 = 17$$

$\Rightarrow$ $2x^2 = 32$ $\Rightarrow$ $x^2 = 16$

$\Rightarrow$ $x = \pm 4$ **(1)**

Hence, for $x = 14$, given pair of determinants is equal.

25. If A is a non-singular matrix of order 3 and $|\text{adj } A| = |A|^k$, then what is the value of k?

All India 2009C; HOTS

We know that, for a square matrix of order n

$|\text{adj}(A)| = |A|^{n-1}$

Here, the order of $A = 3 \times 3$, then $n = 3$

$\therefore$ $|\text{adj}(A)| = |A|^2$ ...(i)

But $|\text{adj}(A)| = |A|^k$ [given] ...(ii)

From Eqs. (i) and (ii), we get

$$k = 2$$ **(1)**

26. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$. Delhi 2009C

$$2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2\,[35-(20)] = 2\,(35-20)$$
$$= 2 \times 15 = 30 \qquad \textbf{(1)}$$

NOTE Suppose we want to multiply with 2 inside the determinant, then we do not multiply each element of determinant. Here, we multiply any one row or column by 2.

27. Find x from equation $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$. All India 2009

Given, $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

$\Rightarrow \quad 2x^2 - 8 = 0 \quad \Rightarrow \quad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$

$\therefore \quad x = \pm 2$ **(1)**

28. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k, if $|2A| = k \cdot |A|$. Foreign 2009

Given, $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ and $|2A| = k \cdot |A|$

$\Rightarrow \quad 2^2 \cdot |A| = k \cdot |A|$

[$\because$ for a square matrix of order 2 $|kA| = k^2 |A|$, k is any scalar]

$\therefore \quad k = 4$ **(1)**

29. Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$. Delhi 2008C

Suppose $A = \begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

On expanding, we get

$A = 2\cos^2\theta - (-2\sin^2\theta)$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$ $\quad [\because \cos^2\theta + \sin^2\theta = 1]$ **(1)**

30. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$. **Delhi 2008; HOTS**

Suppose $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

On expanding, we get

$A = (a+ib)(a-ib) - (c+id)(-c+id)$

$= (a^2 - i^2b^2) - (-c^2 + i^2d^2)$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= (a^2 + b^2) - (-c^2 - d^2)$ $\quad [\because i^2 = -1]$

$= a^2 + b^2 + c^2 + d^2$ **(1)**

31. Find for what value of x, is the following matrix singular?

$\begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix}$ **Delhi 2008**

Do same as Que. 14. **[Ans.** $x = 1$]

32. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, then find the value of x. **Foreign 2008**

Do same as Que. 27. **[Ans.** $x = -13$]